Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	55	35
Section Two: Calculator-assumed	11	11	100	100	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (100 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 7 **(6 marks)**

A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.

(a) Write a rule for the n^{th} term of a sequence in the form $T_n = A + Bn$, which will model this situation where T_n is the amount of water that runs out in the n^{th} minute. (2 marks)

(b) How many litres run out in the seventh minute? (1 mark)

(c) How many litres have run out after eight minutes? (1 mark)

(d) What is the capacity of the tank? (2 marks)

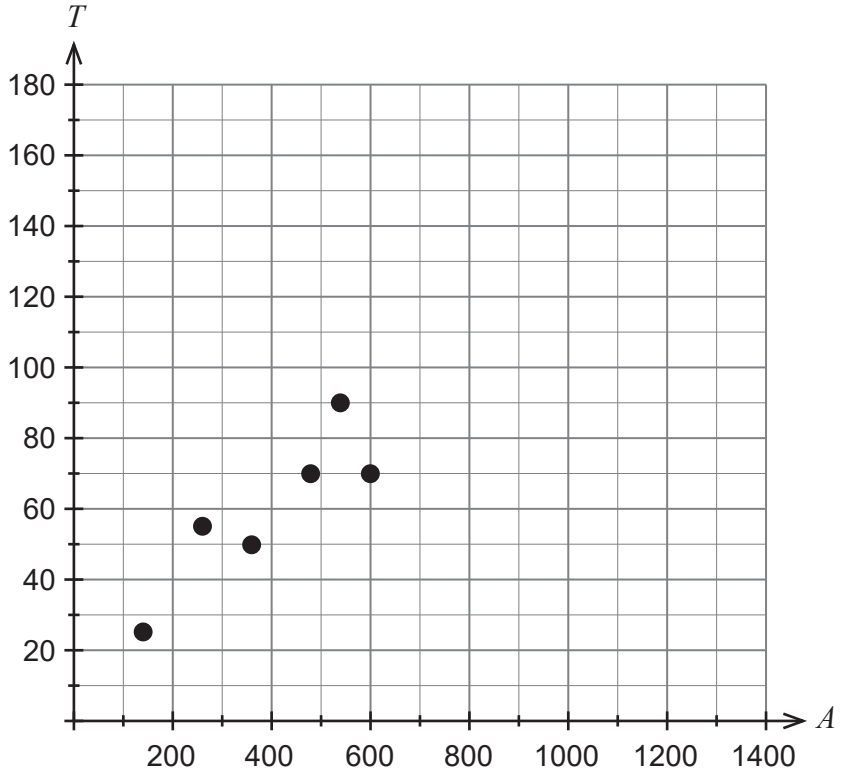
See next page

Question 8 **(13 marks)**

Abdul has a lawnmowing business and is investigating if there is a relationship between the size of a lawn and the length of time it takes to cut the lawn. He takes a random sample of eight customers and measures the areas of their lawns and notes the times, in minutes, it takes to mow their lawns. The results are in the table below, where A is the area of the lawn in square metres and T is the time in minutes. (Note: some values are missing.)

Customer	A	B	C	D	E	F	G	H
A (m^2)		260		480	540	600	860	1180
T (min)	25	55	50	70	90	70	135	140

(a) Complete the scatterplot below. (1 mark)



(b) From the information below, determine the equation of the least-squares line in terms of A and T and state the coefficient of determination for these data. (2 marks)

Linear Reg
$y = ax+b$
a = 0.114691
b = 16.008241
r = 0.9510026
r^2 = 0.9044059

See next page

(c) Interpret the value of the gradient of the least-squares line in the context of the question. (2 marks)

(d) Given that Abdul charges $30 per hour, estimate the charge for mowing a customer's lawn with an area of 500 m^2. (2 marks)

(e) Explain whether the estimate determined in part (d) would be valid. (2 marks)

(f) Using the least-squares line correct to three decimal places

(i) calculate the residuals for Customers B and D. (2 marks)

(ii) explain the significance of the sign and the size of these residuals in reference to the least-squares line. (2 marks)

See next page

Question 9 **(9 marks)**

Corbie and Grant are investigating the possibility of purchasing their own apartment. They will need to open a bank account to save for a deposit. They make an initial contribution of $7000 into the account and add a further $800 at the end of each month. The account has an interest rate of 2.6% per annum, compounded monthly.

(a) How much would be in the account at the end of two years? (3 marks)

Corbie and Grant are willing to pay $280 000 for the apartment and will require an additional $22 000 in fees associated with the purchase. A deposit of 20% of the total cost will be needed in order to qualify for a bank loan.

(b) (i) Show that the required deposit is $60 400. (1 mark)

(ii) How long would it take to save enough for the deposit? (2 marks)

(c) If, at the end of two years, their parents agree to give them a total of $10 000 as a reward for their dedicated saving effort, determine the minimum monthly contribution Corbie and Grant will need to make if they are to have enough for the full deposit after four years. (3 marks)

See next page

Question 10 **(7 marks)**

Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, T, at the end of 2017 was 22 579 and at the end of 2018 was 30 256.

The number of shops operating at the end of n years can be represented by the recursive rule $T_n = 1.34T_{n-1}$, $T_1 = 15$.

(a) Show mathematically that the common ratio is approximately 1.34. (1 mark)

(b) Write the rule for the n^{th} term of this sequence. (1 mark)

(c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. (2 marks)

Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, $114.80 per day.

(d) Calculate the total daily wages for all stores at the beginning of 2012. (3 marks)

See next page

Question 11 **(13 marks)**

Data for the total occupancy of rooms for each season of the year at a Perth hotel is shown below.

n	Year	Season	Total rooms occupied	Seasonal mean	4-point centred moving average	Total rooms occupied as a percent of seasonal mean
1	2015/16	Spring	1770	1660.5		106.59
2		Summer	1904			**B**
3		Autumn	1591		1644.375	95.81
4		Winter	1377		1622.5	82.93
5	2016/17	Spring	1641	1610.25	1618	101.91
6		Summer	1858		1614.75	115.39
7		Autumn	1601		1602.25	99.43
8		Winter	1341		1584.75	83.28
9	2017/18	Spring	1577	1524.0	1558	103.48
10		Summer	**A**		1532.375	116.93
11		Autumn	1463		1526.875	96.00
12		Winter	1274		1525.125	83.60
13	2018/19	Spring	1600	1519.75	**C**	105.28
14		Summer	1745		1525.25	114.82
15		Autumn	1504			98.96
16		Winter	1230			80.93

(a) Calculate the value of **A**, **B** and **C**. (3 marks)

See next page

(b) Complete the table showing the seasonal index for each season. (1 mark)

Summer	Autumn	Winter	Spring
1.1545		0.8268	1.0432

(c) Calculate the deseasonalised value for Winter 2017/18. (2 marks)

(d) Comment on the effect the seasonal index had on the value found in part (c). (1 mark)

(e) The least-squares line using deseasonalised data is $R = -12.071n + 1681.25$. Use this line to predict the total number of rooms occupied during Spring 2020/21. (2 marks)

When a prediction was made for Spring 2020/21, using the least-squares line based on the 4-point centred moving averages, the answer was 1481.

(f) Explain why this is different from the answer obtained in part (e). (1 mark)

The manager of the hotel attended a meeting with the owners of the hotel. She explained to the owners that the reduction in occupancy was due to the downturn in the Western Australian economy in recent years.

(g) Comment on the statement made by the hotel manager. (2 marks)

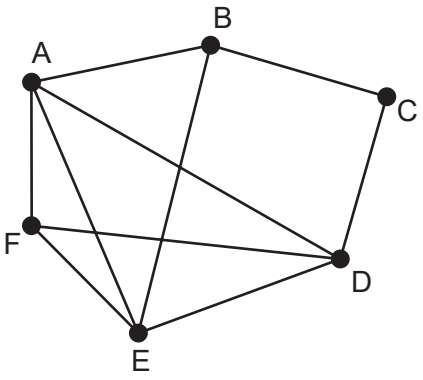
(h) What practical advice, in the context of the question, would you give to the manager of the hotel? (1 mark)

Question 12 **(6 marks)**

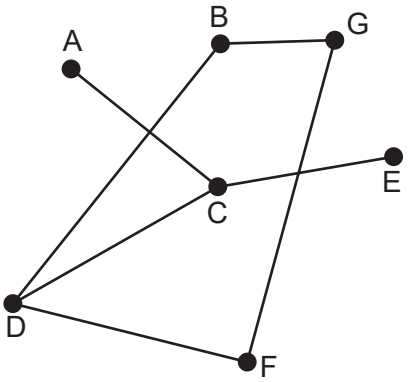
Jake, a park ranger, is giving a presentation at a National Park and Wildlife Conference on possible designs for a new park. Unfortunately, Jake made mathematical errors in his presentation about the paths (represented by edges) and shelter huts (represented by vertices) in the park.

(a) For each of the following statements, the graph drawn by Jake was incorrect. Redraw the graph to match the statement correctly.

(i) This park plan has been drawn as a connected planar graph containing six vertices. (2 marks)



(ii) This park plan has been drawn as a bipartite graph. (3 marks)



Jake also makes the following incorrect statement in his presentation. ‘A park plan can be a complete graph with 21 paths and six shelter huts’.

(b) If the plan must be a complete graph with 21 paths, how many shelter huts should Jake have quoted? (1 mark)

See next page

Question 13 **(10 marks)**

Mehmet has saved $3600 from wages received at a part-time job. He is keen to invest this money in an account which earns 3.65% per annum, compounded monthly.

Over the next three years, Mehmet plans to continue working part-time and is aiming to make deposits of $250 at the end of each month.

(a) Write a recursive relation to give the value of the investment at the end of each month. (2 marks)

(b) Mehmet hopes that this investment will double his initial savings in one year. Justify whether this is possible. (2 marks)

(c) Determine the total amount of interest Mehmet would receive after three years. (3 marks)

Unfortunately, after two years, Mehmet's working hours are reduced and he is only able to deposit $120 at the end of each month.

(d) By how much would this reduce the value of his investment by the end of the three years? (3 marks)

See next page

Question 14 **(10 marks)**

The table below contains data provided by the Australian Bureau of Statistics. It shows the number of households with and without internet access from 2014–2017. All values are in thousands of households.

Internet access

State/territory	2014–15			2016–17		
	Households with internet access '000	Households without internet access '000	Total '000	Households with internet access '000	Households without internet access '000	Total '000
New South Wales	2407.9	414.5	2822.4	2439.9	421.8	2861.7
Victoria	1934.2	305.1	**A**	2008.2	305.8	2314.0
Queensland	1552.4	248.5	1800.9	1591.9	249.8	1841.7
South Australia	565.1	121.4	686.5	575.5	**B**	696.6
Western Australia	843.6	113.0	956.6	859.7	112.6	972.3
Tasmania	172.0	38.7	210.7	177.7	36.2	213.9
Northern Territory	58.1	6.3	64.4	57.6	7.3	64.9
Australian Capital Territory	137.2	9.0	146.2	140.1	9.7	149.8
Total	7670.5	1256.5	8927.0	7850.6	1264.3	9114.9

(a) (i) Determine the value of **A** and **B** in the table above. (2 marks)

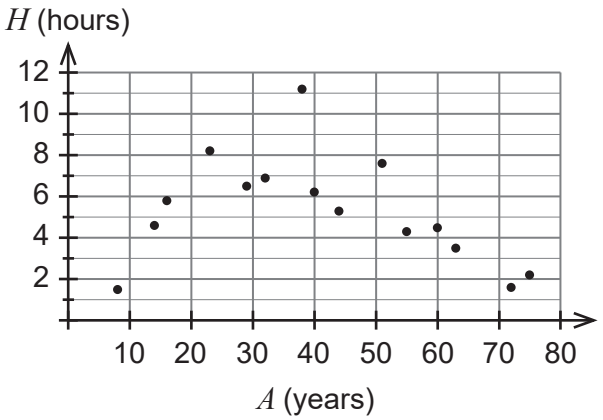
(ii) Compare the percentages, correct to two decimal places, of households with internet access in New South Wales between 2014–15 and 2016–17. Comment on your results. (3 marks)

See next page

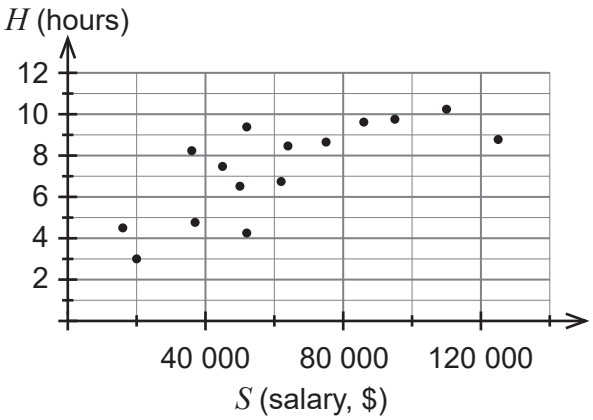
(iii) What is the difference in the data for households with internet access for the Northern Territory over the time period shown, compared to other States and Territories? (1 mark)

An internet service provider from Tasmania wanted to determine whether a person's age, A, or salary, S, affected the number of hours, H, of internet usage per day. The graphs below each show the recorded data for people surveyed.

Daily Internet Usage According to Age



Daily Internet Usage According to Salary



(b) (i) Describe the association between a person's salary and the number of hours of internet usage per day, in terms of direction and form. (2 marks)

(ii) The internet service provider calculated the correlation coefficient for the data contained in each graph. The values they calculated are contained in the following list.

–1.25, –0.95, –0.75, –0.3, 0.1, 0.3, 0.75, 0.95, 1.25

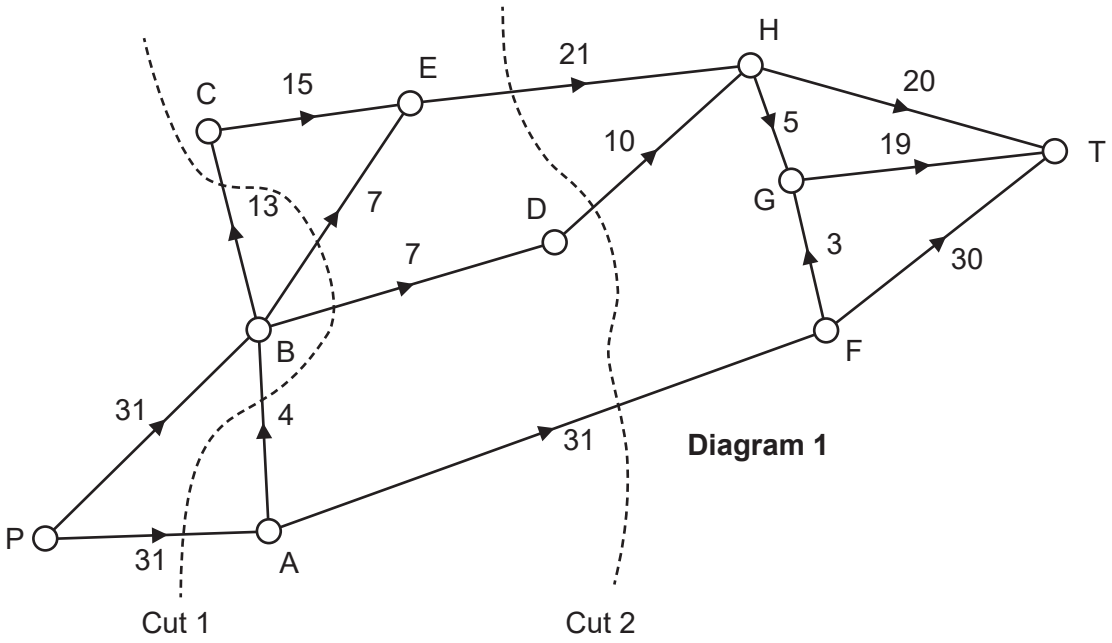
Choose the best estimate from the list for each of the graphs shown above. (2 marks)

Graph	Correlation coefficient
Daily internet usage according to age	
Daily internet usage according to salary	

See next page

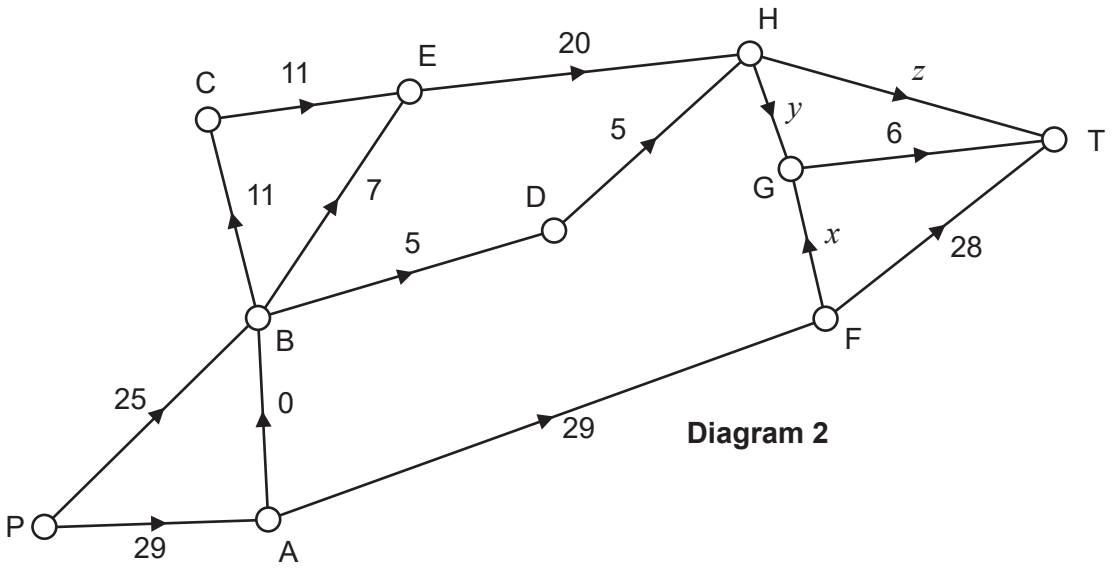
Question 15 **(11 marks)**

The directed network below shows the maximum available capacity for transferring power between different sub-stations on a small island. The number on each edge gives the capacity in kilovolts (kV).



(a) State the capacity of each cut in Diagram 1. (2 marks)

Diagram 2 shows a possible flow through the same network.



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(b) Determine the initial flow in Diagram 2. (1 mark)

(c) Calculate the value of x, y and z in Diagram 2. (3 marks)

(d) Determine the maximum flow for the original network (Diagram 1). (2 marks)

Engineers wish to increase the maximum capacity to sub-station T. They propose to add a new transmission line from E to T of capacity 3 kV **or** a new transmission line from D to G of capacity 3 kV.

(e) Determine which of these proposals will increase the maximum capacity to sub-station T. Justify your answer. (3 marks)

Question 16 **(7 marks)**

The table below records the altitude (metres above sea level), latitude (° S) and mean maximum temperature (°C) during January for eight cities in the southern hemisphere.

Altitude (A)	Latitude (L)	Mean maximum temperature (T)
15	31.95	25
20	43.53	20
24	42.88	18
314	45.03	16
8	6.18	28
154	12.05	26
37	12.46	29
8	34.60	25

Comparing altitude and the mean maximum temperature, it was determined that the least-squares line for these data was $T = -0.022A + 24.97$ and $r_{AT} = -0.50$.

(a) Determine the coefficient of determination for altitude and the mean maximum temperature and interpret this value. (2 marks)

(b) Determine the equation of the least-squares line for comparing latitude and the mean maximum temperature and state the correlation coefficient. (2 marks)

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice. (3 marks)

See next page

Question 17 **(8 marks)**

Joel has set up a special investment fund that has a current balance of $350 000. He contributes 7.5% of his monthly income to the investment and has an overseas pension which contributes a further $355 per month. The investment fund has an interest rate of 6.5% per annum, compounded monthly. Joel's annual salary is $101 000 and he has just turned 60 years of age.

(a) Calculate Joel's total monthly contribution to the fund. (2 marks)

(b) Calculate the lump sum that he could receive if he retires on his 67th birthday. (2 marks)

Joel retires at 67 and wants to use his lump sum payment to set up a regular income. He decides to look at two options that offer monthly payments.

Option 1: A reducing balance annuity at 7% per annum, compounded monthly.
Option 2: A perpetuity at 7.5% per annum, compounded monthly.

(c) Calculate his maximum monthly income for the next 20 years using Option 1. (2 marks)

(d) Calculate his monthly income using Option 2. (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

ACKNOWLEDGEMENTS

Question 14 Adapted from: Australian Bureau of Statistics. (2018, March 28). *8146.0 - Household Use of Information Technology, Australia, 2016-17* and *8146.0 - Household Use of Information Technology, Australia, 2014-15.* Retrieved May, 2019, from http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/8146.02016-17?OpenDocumentbureau
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